# Practical models of solar energetic particle transport

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***Abstract*. Impulsive solar energetic particle (SEP) events are associated with corotating solar wind structures, while gradual SEP events are associated with coronal mass ejections (CMEs) and their continuation in the interplanetary space. Those solar wind structures strongly affect the SEP transport to a spacecraft. This report is focused on new SEP modeling schemes, which are applicable to a general case of the energetic particle propagation in dynamic, structured solar wind. Accurate analysis of a SEP event should incorporate modeling of the particle transport in structured solar wind, which can be done with use of coronal and solar wind data, MHD modeling of solar wind, the SEP flux anisotropy measurements, and the new SEP modeling schemes described in this report.**

***Keywords*: Energetic solar particles – propagation of energetic particles – particle acceleration**

## I. Introduction

Numerical modeling of solar energetic particle (SEP) events helps us to understand what we observe in space, to uncover physics of the SEP sources, and to probe interplanetary magnetic field (IMF) structures met by SEPs (e.g., [1], [2], [3]). Many previous works (e.g., [4], [5], [6]) considered an Archimedean spiral magnetic field configuration, as expected for a uniform solar wind due to the rotation of the Sun. However, even for weak impulsive SEP events it is essential to take into account deviations from the standard IMF [7], [8]. Major events frequently occur as part of a series of eruptions from one active region, so it is not unusual for a gradual SEP event to occur in strongly disturbed solar wind. Recent modeling efforts were focused on particle transport in disturbed magnetic field configurations, such as corotating compression regions [9], [10], open tubes with magnetic bottlenecks [11] and interplanetary magnetic clouds [12], [13].

While the mean free path of energetic particles in the inner heliosphere is highly variable, its value is often comparable to or even exceeds 1 AU, which necessitates a focused transport approach for the SEP modeling. Ruffolo [4] considered the focused transport of solar cosmic rays with both pitch-angle scattering and energy change included, based on the assumption that the large-scale structure of the interplanetary magnetic field is stationary in a frame corotating with the Sun, and the solar wind flow in that frame is parallel to the magnetic field (Fig. 1). Such assumptions are valid for the Archimedean spiral magnetic field and for some other corotating structures [9]. However, a dynamic solar wind model that is based on observations of a particular event may not fulfill these assumptions. Modeling of energetic particle transport in arbitrary solar wind structure requires a general solution to the focused transport problem of SEPs.

In an effort to combine SEP transport models with results of MHD modeling of CMEs, Kóta *et al.* [14] employed the field-aligned SEP-transport equation in the non-inertial frame co-moving with the solar wind plasma at every position of an energetic particle [15], [16]. Recently, Kocharov *et al.* [17] have formulated a method for stochastic simulations of SEP transport in an arbitrarily structured solar wind with use of only inertial reference frames (see also Section III).

## II. Stochastic Simulations of SEP transport

The evolution of energetic particle distributions undergoing energy and pitch angle changes can be treated by the method of stochastic simulations, known also as the Monte Carlo method. There are two slightly different approaches to the Monte Carlo method. The first approach is based on the fact that the Fokker-Planck equation can be expressed as a set of stochastic differential equations with random scattering terms (e.g., [18] and references therein). The set can be solved by time-stepping the progress of individual (quasi-)particles, and the results – e.g., particles with a particular energy at a particular position – are binned to give a final distribution. The formal solving of transport equation with Monte Carlo method may simulate not real physical processes with real particles but imaginary particles in imaginary processes (e.g., [19]). However, it is also possible to start with a microscopic description of physical processes, to develop a numerical code, and afterwards if possible to link the code to a Fokker-Planck equation. The stochastic simulations method was applied in particular to modeling of nuclear interactions of high-energy particles in solar flares (e.g., [20], [21]) and to SEP transport in interplanetary space (e.g., [22], [6]). The Monte Carlo method can also incorporate ionization and recombination of accelerated ions, the generation of plasma waves by energetic protons and electrons, and any other elementary processes in question (e.g., recent [23], [24]).

Ruffolo [4] derived a Fokker-Plank equation for the transport of solar energetic particles in the interplanetary magnetic field, assuming that in a frame corotating with the Sun the large-scale structure of the field is stationary, and particles are scattered at irregularities in the solar wind flow along the large-scale magnetic field lines. In the case of a static magnetic field and field-aligned solar wind flow, both the particle transport and the energy change are completely due to concurrent focusing by the IMF and scattering in the plasma stream. In such a case the Monte Carlo simulations in the corotating frame comprise three small-step processes – particle motion along the IMF line, particle focusing in the static magnetic field, and pitch-angle scattering at scattering centers in the flow [6]. We calculate, at each Monte-Carlo time step, $\delta t$, the change of the particle coordinate due to the particle motion along the magnetic field line as

$$\delta\xi = v\mu\delta t, \quad (1)$$

and the change of the particle pitch angle cosine due to focusing and scattering:

$$\delta\mu = (1-\mu^2)v\delta t/(2L) + \delta\mu_{\rm sc}, \quad (2)$$

where $\xi$ is particle coordinate measured along the magnetic field line from the Sun, $v$ and $\mu$ are the particle speed and pitch angle cosine, respectively;

$$L = -\frac{B}{\partial B/\partial\xi} \quad (3)$$

is the magnetic focusing length, $B$ is the magnetic field intensity, and $\delta\mu_{\rm sc}$ is a random term caused by angular scattering with scattering frequency

$$\nu = \nu(\mu). \quad (4)$$

The scattering can be accounted for by a process of 3−D angular scattering with respect to the instantaneous direction of particle motion, as described in [25]. Scattering of energetic particles at magnetic irregularities tends to isotropize the particle distribution in the rest frame of plasma, which is a 'natural' reference frame for simulations of scattering, and the scattering frequency $\nu$ and the mean free path $\lambda$ are defined in that frame. In the case of the standard solar wind we can perform scattering in a local frame that moves with respect to the corotating frame with solar wind velocity $u_c = u/\cos\Psi$, where $\Psi$ is angle between the magnetic field and the radial direction (Fig. 1). Note that the 3−D angular scattering approach avoids possible erroneous treatment of particle distributions at $\mu = \pm 1$ encountered by direct stochastic simulations in $\mu$. In the case of the standard solar wind the results of Monte Carlo simulations of SEP transport have been confirmed with a finite-difference solution of [4] to three-digit accuracy [6].

## III. General Solution to Focused Transport Problem

In the general case, there is no global frame where the solar wind flow is parallel to the large-scale magnetic field and the magnetic field is static. Correspondingly, stochastic simulations of particle transport must be modified to incorporate the cross-field flow of the solar wind plasma with frozen-in magnetic field lines as well as

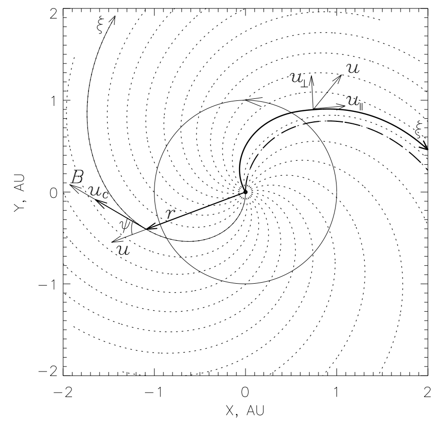


Fig. 1: A corotating solar wind structure employed for particle transport modeling [9], [10], [17] and vectors used for general formulations. Solar wind velocity vectors in the inertial frame and in the corotating frame are $\boldsymbol{u}$ and $\boldsymbol{u}_c$, respectively.

possible explicit time-dependence of the magnetic field. After consideration of the underlying physics, a general SEP modeling method can be formulated.

We will use for modeling of physical processes only inertial reference frames. Particle motion between the collisions, magnetic focusing, and energy change due to centripetal force on moving magnetic field lines (to be introduced below) will be modeled in the global, fixed inertial frame. The same frame will be used for registering of particle parameters at 1 AU. For the particle's position, we use a curvilinear coordinate measured along the fixed magnetic field line. Scattering and explicit betatron effect (see below) will be modeled in a local inertial frame (instantly) co-moving with solar wind at a current time and at a current position. We do not use the non-inertial co-moving frame employed in [14]. A solar-corotating-frame modeling can be used for testing of the inertial-frame modeling in the case of the system shown in Fig. 1, with particles registered in the fixed inertial frame for both modeling methods [17].

Our simulations of the particle's displacement along the IMF line with velocity $v\mu$, focusing and scattering in the case of arbitrary solar wind, are basically similar to those described by equations (1)–(4) for the standard solar wind; however, our simulations are performed in an inertial heliocentric frame of reference. Inasmuch as both the solar wind plasma with irregularities and the SEPs are anchored to the IMF lines, the relative motion of SEPs and plasma depends only on the component of the plasma velocity parallel to the magnetic field, $\boldsymbol{u}_\parallel$ (Figure 1). Therefore, the velocity $u_\parallel$ is used instead of $u_c$ for simulations of scattering. Then we introduce three additional processes associated with the solar wind flow in the direction perpendicular to IMF, $u_\perp \neq 0$, and with explicit time dependence of the magnetic field intensity, $\partial B/\partial t \neq 0$.

The first process is *transverse convection* of SEPs with magnetic field lines. Particle transport in the direction perpendicular to the magnetic field is assumed to proceed with the perpendicular velocity of the solar wind, $\boldsymbol{u}_\perp$. We use for Monte Carlo simulations a curvilinear coordinate, $\xi$, that is measured along a magnetic field line. The coordinate line can move with the solar wind flow. In such a coordinate system, the particle's coordinate change is the difference between the particle's shift due to convection and the shift of the coordinate grid. Thus, at each time step, $\delta t$, we add to the particle's coordinate a displacement $\delta\xi_{\rm pc}$ that is the difference between the displacement $\boldsymbol{u}_\perp\delta t$ and the displacement of the coordinate grid. For instance, in a corotating structure like that shown in Figure 1, the azimuthal component of convection does not change the particle's coordinate $\xi$ measured along the magnetic field line from the Sun. A $\xi$-coordinate change is caused only by a radial displacement, $(\boldsymbol{u}_\perp)_r\,\delta t$:

$$\delta\xi_{\rm pc} = \frac{(\boldsymbol{u}_\perp)_r\,\delta t}{\cos\Psi} = u\sin\Psi_u\tan\Psi\;\delta t, \qquad (5)$$

where the radial component of the perpendicular velocity of solar wind $(\boldsymbol{u}_\perp)_r = u\sin\Psi_u\sin\Psi$, $\Psi$ is the angle between the magnetic field and the radial direction, and $\Psi_u$ is the angle between the magnetic field and the solar wind velocity. Accounting for the perpendicular convection, equation (1) is replaced with:

$$\delta\xi = v\mu\delta t + u_\psi\delta t, \qquad (6)$$

where

$$u_\psi = u\sin\Psi_u\tan\Psi \qquad (7)$$

is the explicit convection speed of SEPs due to $u_\perp$. The introduced vectors $(\boldsymbol{u}_\perp)_r$ and $\boldsymbol{u}_\psi$ are illustrated in Fig. 1 where $\Psi_u = \Psi$. There is also a parallel convection due to $u_\parallel$, which is implicitly accounted for, with particle scattering being performed in the frame moving with velocity $u_\parallel$.

The second process is *explicit betatron effect* (deceleration or acceleration) caused by the perpendicular convection. This effect appears when the magnetic field intensity $B$ changes in a direction perpendicular to the magnetic field. The betatron effect can change the component of particle velocity perpendicular to the magnetic field, so that at each step the perpendicular velocity squared changes by

$$\delta v_\perp^2 = v_\perp^2\frac{1}{B}\,\boldsymbol{u}_\perp\cdot\nabla B\;\delta t \qquad (8)$$

The betatron effect may be also caused by explicit time dependence of the evolving magnetic field intensity, which results in

$$\delta v_\perp^2 = v_\perp^2\frac{1}{B}\frac{\partial B}{\partial t}\delta t. \qquad (9)$$

Both contributions can be combined as

$$\delta v_\perp^2 = v_\perp^2\frac{1}{B}\frac{d_\perp B}{dt}\delta t, \qquad (10)$$

where we have defined the perpendicular material derivative as

$$\frac{d_\perp}{dt} = \frac{\partial}{\partial t} + \boldsymbol{u}_\perp\cdot\nabla \qquad (11)$$

It is convenient to introduce a dimensionless rate coefficient for the explicit betatron effect, $\alpha_b$:

$$\delta v_\perp^2 = \alpha_b v_\perp^2\frac{\delta t}{R_o/U_o}\,, \qquad \alpha_b \equiv \frac{1}{B}\frac{d_\perp B}{dt}\frac{R_o}{U_o}\,, \qquad (12)$$

where one can adopt $U_o = 400$ km s$^{-1}$ and $R_o = 1$ AU.

The third process is the change in a particle's parallel velocity on a curved magnetic field line due to motion of the field line. This is similar to the energy change of a particle bouncing from a moving body. Change in the particle's parallel energy is caused by the work done by the particle against the centripetal force, as the particle moves along a curved IMF line, which in turn moves with the plasma velocity $\boldsymbol{u}_\perp$ in the direction perpendicular to the field line. The work done by the centripetal force is $\delta W = mv_\parallel^2\boldsymbol{R}_c^{-1}\cdot\boldsymbol{u}\,\delta t$, where the vector of inverse radius of curvature $\boldsymbol{R}_c^{-1} = \partial\boldsymbol{b}/\partial\xi$ and $\boldsymbol{b} \equiv \boldsymbol{B}/B$ is a unit vector that is parallel to magnetic field line. Thus, the change of the parallel component of a particle's velocity due to the *centripetal effect* is of the form:

$$\delta v_\parallel = v_\parallel\,\boldsymbol{u}\cdot\frac{\partial\boldsymbol{b}}{\partial\xi}\,\delta t = v_\parallel\,\boldsymbol{u}_\perp\cdot\frac{\partial\boldsymbol{b}}{\partial\xi}\,\delta t. \qquad (13)$$

After defining one more rate coefficient, $\alpha_c$, we finally have:

$$\delta v_\parallel = \alpha_c v_\parallel\frac{\delta t}{R_o/U_o}\,, \qquad \alpha_c \equiv \boldsymbol{u}_\perp\cdot\frac{\partial\boldsymbol{b}}{\partial\xi}\frac{R_o}{U_o}. \qquad (14)$$

Here, we have neglected a time-change of the magnetic field direction, assuming $|\partial\boldsymbol{b}/\partial t| \ll v_\parallel|\partial\boldsymbol{b}/\partial\xi|$, otherwise one has to replace equation (13) with

$$\delta v_\parallel = v_\parallel\,\boldsymbol{u}_\perp\cdot\frac{\partial\boldsymbol{b}}{\partial\xi}\,\delta t + \frac{\mu}{|\mu|}\,\boldsymbol{u}_\perp\cdot\frac{\partial\boldsymbol{b}}{\partial t}\delta t. \qquad (15)$$

Processes (12) and (14) change both a particle's energy and its pitch angle. Profiles of three new coefficients, $u_\psi/U_o$, $\alpha_b$ and $\alpha_c$, can be calculated along a particular magnetic field line and used for simulations of particle propagation in the general case when the solar wind flow is not parallel to the magnetic field and the magnetic field is not static.

## IV. Connection to Diffusion-Convection Equation

The diffusion-convection equation (DCE) provides the analytic background for transport and acceleration of cosmic rays where the high-energy particle distribution is nearly isotropic in the local plasma flow frame ([22] and references therein):

$$\frac{\partial f}{\partial t} = \nabla_i\kappa_{ij}\nabla_j f - \boldsymbol{u}\cdot\nabla f + \frac{p}{3}\frac{\partial f}{\partial p}\,\nabla\cdot\boldsymbol{u}. \qquad (16)$$

Here the (isotropic) distribution function $f$ is the particle number density in six-dimensional momentum-coordinate space ($p_i$, $x_i$; $i = 1, 2, 3$). Representing the isotropic part of particle distribution, the function $f$ depends only on magnitude $p$ of the momentum vector $p_i$. The first term on the right-hand side of equation (16) comprises scattering and magnetic geometry effects, including focusing and drifts. The second term represents convection with the plasma flow velocity, $\boldsymbol{u}$, and the third term accounts for the compressive deceleration/acceleration, depending on the sign of the divergence of the fluid velocity, $\nabla \cdot \boldsymbol{u}$. Equation (16) suggests that the kinetic energy of nonrelativistic particles, $E$, in an expanding system ($\nabla \cdot \boldsymbol{u} > 0$) declines as

$$\frac{dE}{dt} = -\frac{2}{3} E \, \nabla \cdot \boldsymbol{u}. \tag{17}$$

One can re-cast the energy-changing term using MHD equations [10]. As the magnetic field is frozen into the plasma, the magnetic field is linked to the fluid velocity by the MHD induction equation:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}). \tag{18}$$

Multiplying this equation by $\boldsymbol{b}/|\boldsymbol{B}|$, where $\boldsymbol{b} \equiv \boldsymbol{B}/|\boldsymbol{B}|$ is a unit vector parallel to magnetic field, after straightforward vector algebra one can express the divergence of solar wind velocity in the following form:

$$\nabla \cdot \boldsymbol{u} = \boldsymbol{b} \cdot (\boldsymbol{b} \cdot \nabla) \, \boldsymbol{u} - \frac{1}{B} \frac{dB}{dt}, \tag{19}$$

where $dB/dt = \partial B/\partial t + (\boldsymbol{u} \cdot \nabla) B$ is the material derivative of the magnetic field magnitude $B \equiv |\boldsymbol{B}|$. Then, after splitting $\boldsymbol{u} = \boldsymbol{u}_\parallel + \boldsymbol{u}_\perp$ the divergence of plasma velocity can be recast into the form:

$$\nabla \cdot \boldsymbol{u} = (\boldsymbol{b} \cdot \nabla) \, u_\parallel - \frac{1}{B} \left( \boldsymbol{u}_\parallel \cdot \nabla \right) B - \frac{1}{B} \frac{d_\perp B}{dt} - \boldsymbol{u}_\perp \cdot \frac{\partial \boldsymbol{b}}{\partial \xi}, \tag{20}$$

where $d_\perp B/dt$ is defined by equation (11). The two last terms arise from the perpendicular convection and explicit time dependence of the magnetic field intensity. Substituting those terms into equation (17), one can find the associated energy-change rate, $(dE/dt)_\perp$. With definitions (12) and (14), the rate of energy change associated in DCE with $\boldsymbol{u}_\perp$ and $\partial B/\partial t$ is of the form:

$$\left( \frac{dE}{dt} \right)_\perp = \frac{2}{3} \left( \alpha_b + \alpha_c \right) \frac{U_o}{R_o} E. \tag{21}$$

On the other hand, one can start with the microscopic processes defined in the previous section and average over an isotropic particle distribution the energy change defined by equations (12) and (14):

$$m \left( \frac{1}{2} \langle \delta v_\perp^2 \rangle + \langle v_\parallel \delta v_\parallel \rangle \right), \tag{22}$$

to obtain an expression identical to equation (21). The other terms of equation (20) can also be explained from a microscopic point of view. The first term in the right-hand-side of equation (20) arises from the first-order Fermi-type acceleration/deceleration as particles scatter at inhomogeneities that flow with different velocities at different locations. The second term is due to the betatron effect in the local frame that moves with the fluid velocity $u_\parallel$. Thus, there is a one-to-one correspondence between the processes introduced in Monte Carlo simulations and the components of the energy-changing term in the diffusion-convection equation.

In addition to the field-aligned particle motion, a cross-field particle transport has been also incorporated into the focused transport models [26], [27] and can be used when it is justified by the SEP data and the interplanetary magnetic field measurements.

Accurate measurements of the particle pitch-angle distributions in some SEP events indicate that current scattering models may be not always valid [28], while in other events the quasilinear theory gives satisfactory fits to the data [1].

## References


[1] L. Kocharov, O. Saloniemi, J. Torsti, G. Kovaltsov, E. Riihonen (2007) *ApJ*, 654, 1121.
[2] A. Sáiz, D. Ruffolo, J. W. Bieber, P. Evenson, R. Pyle (2008) *ApJ*, 672, 650.
[3] N. Agueda, R. Vainio, D. Lario, B. Sanahuja (2008) *ApJ*, 675, 1601.
[4] D. Ruffolo (1995) *ApJ*, 442, 861.
[5] R. Hatzky, M.-B. Kallenrode, J. M. Schmidt (1997) *Proc. 25th Internat. Cosmic Ray Conf. (Durban)*, 1, 245.
[6] L. Kocharov, R. Vainio, J. Torsti, G. A. Kovaltsov (1998) *Solar Phys.*, 182, 195.
[7] L. Kocharov, V. J. Pizzo, R. D. Zwickl, E. Valtonen (2008) *ApJL*, 680, L69.
[8] L. Kocharov, J. Laivola, G. M. Mason, L. Didkovsky, D. L. Judge (2008) *ApJS*, 176, 497.
[9] J. Giacalone, J. R. Jokipii, J. Kóta (2002) *ApJ*, 573, 845.
[10] L. Kocharov, G. A. Kovaltsov, J. Torsti, A. Anttila, T. Sahla (2003) *J. Geophys. Res.*, 108, A11, 1404.
[11] J. W. Bieber, W. Dröge, P. A. Evenson, R. Pyle, D. Ruffolo, U. Pinsook, P. Tooprakai, M. Rujiwarodom, T. Khumlumlert, S. Krucker (2002) *ApJ*, 567, 622.
[12] L. Kocharov, G. A. Kovaltsov, J. Torsti, K. Huttunen-Heikinmaa (2005) *J. Geophys. Res.*, 110, A12S03.
[13] D. Ruffolo, P. Tooprakai, M. Rujiwarodom, T. Khumlumlert, M. Wechakama, J. W. Bieber, P. Evenson, R. Pyle (2006) *ApJ*, 639, 1186.
[14] J. Kóta, W. B. Manchester, J. R. Jokipii, D. L. de Zeeuw and T. I. Gombosi (2005) *The Physics of Colissionless Shocks, AIP Conf. Proc.*, 781, 201.
[15] J. Skilling (1971) *ApJ*, 170, 265.
[16] P. A. Isenberg (1997) *J. Geophys. Res.*, 102, A3, 4719.
[17] L. Kocharov, V. J. Pizzo, D. Odstrcil, R. D. Zwickl (2009) *J. Geophys. Res.*, 114, A05102.
[18] L. Fletcher (1994) *Space Sci. Rev.*, 68, 239.
[19] G. Qin, M. Zhang, J. R. Dwyer (2006) *J. Geophys. Res.*, 111, A08101.
[20] X.-M. Hua and R. E. Lingenfelter (1987) *Solar Phys.*, 107, 351.
[21] N. Mandzhavidze and R. Ramaty (1992) *ApJ*, 389, 739.
[22] I. N. Toptygin (1985), *Cosmic rays in interplanetary magnetic fields (Geophysics and Astrophysics Monographs, No 27)*, D. Reidel, Dordrecht.
[23] Y. Y. Kartavykh, W. Dröge, B. Klecker, G. M. Mason, E. Mbius, M. Popecki, S. Krucker (2007) *ApJ*, 671, 947.
[24] R. Vainio and T. Laitinen (2007) *ApJ*, 658, 622.
[25] L. Kocharov, J. Torsti, T. Laitinen, R. Vainio (1997) *Solar Phys.*, 175, 785.
[26] M. Zhang, G. Qin, H. Rassoul (2009) *ApJ*, 692, 109.
[27] W. Dröge, Y. Kartavykh, B. Klecker, G. A. Kovaltsov (2009) *Proc. 31st ICRC*, report 244.
[28] W. Dröge and Y. Y. Kartavykh (2009) *ApJ*, 693, 69.